Magnetic Field of Helical Conductors with Finite Length

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Abstract – Transmission of the electric power is accompanied with generation of low – frequency electromagnetic fields. Electromagnetic compatibility studies require that the fields from sources of electric power be well known. Unfortunately, many of these sources are not defined to the desired degree of accuracy. This applies e.g. to the case of the twisted-wire pair used in telephone communication; already practiced is twisting of insulated high-voltage three phase power cables and single-phase distribution cables as well.

The paper presents a theoretical study of the calculation of magnetic fields in vicinity of conductors having helical structure. For the helical conductor with finite length the method is based on the Biot-Savart law. Since the lay-out of the cables is much more similar to a broken line than to strait line, in the paper the magnetic flux densities produced by helical conductor of complex geometry are also derived.

The analytical formulas for calculating the 3D magnetic field can be used by a software tool to model the magnetic fields generated by e.g. twisted wires, helical coils, etc.

I. MAGNETIC FIELD CALCULATION

An analytical method for calculating the low-frequency magnetic field of an infinitely long helical line current using the magnetic vector potential has been derived in the pioneering work [1] and the problem has afterwards been revisited in [2] – [6], [8].

The realistic model of twisted cables should be however based on the theory of a helical line current of finite length instead on the theory of infinitely long one. Moreover, the lay-out of the cables is much more similar to a broken line than to strait line. In the paper the magnetic flux density produced by finite length helical conductor of complex geometry is derived. It is assumed, that the currents induced in the earth can be neglected, so the magnetic field can be obtained using the Biot-Savart law:

$$\vec{B}(x,y,z) = \frac{\mu_0}{4\pi}\int_c \frac{\underline{I}(\vec{dl}\times 1_r)}{r^2} \qquad (1)$$

where $\underline{I}$ is a phasor current, the vector element $\vec{dl}$ coincides with the direction of the current I, 1_r is a unit vector in the direction of the vector $\vec{r}$, r is the distance between the source point and the observation point and μ_0 is the magnetic permeability of the vacuum.

The analytical formulas for calculating the 3D magnetic field with respect to a convenient and unique reference system are derived. For calculation purposes, the helix route is divided into straight segments. For simplicity consider only the i-th segment of the helix. It is convenient to define two different Cartesian reference systems: the first one x,y,z is a reference system (external reference system), the second one x',y',z' is referred to the i-th segment, Fig. 1. It should be noted, that the reference coordinate system (unprimed) can be arbitrary located in the space.

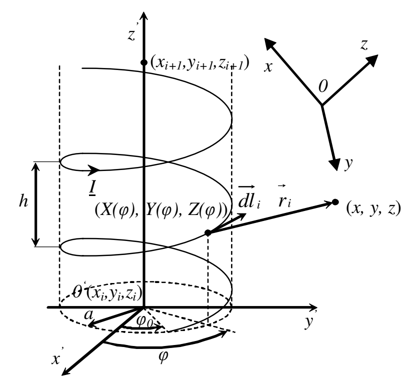

Fig.1. Reference systems and the i-th segment of the helix

The point of origin of the primed coordinate system $0'$ (outset of the helix axis) have in the external (unprimed) reference system the coordinates (x_i,y_i,z_i), whereas the end point of the i-th segment axis has the coordinates $(x_{i+1},y_{i+1},z_{i+1})$, respectively.

The parametric equations of the helical line with respect to the parameter φ $(0 \le \varphi \le 2\pi L_i / h)$ indicated on Fig.1 and with $\varphi_0 = 0$ are:

$$X'(\varphi) = a\cos\varphi,\; Y'(\varphi) = a\sin\varphi,\; Z'(\varphi) = \frac{h}{2\pi}\varphi \qquad (2)$$

where a is the helix radius, h means the helix pitch, L_i is the length of the i-th helix segment.

To obtain the eqn.(2) in the reference coordinates system, the roto-translation formulas in the tridimensional space should be applied. Thus:

$$\begin{bmatrix} X(\varphi) \\ Y(\varphi) \\ Z(\varphi) \end{bmatrix} = \begin{bmatrix} \alpha_{1i} & \beta_{1i} & \gamma_{1i} \\ \alpha_{2i} & \beta_{2i} & \gamma_{2i} \\ \alpha_{3i} & \beta_{3i} & \gamma_{3i} \end{bmatrix} \begin{bmatrix} X'(\varphi) \\ Y'(\varphi) \\ Z'(\varphi) \end{bmatrix} + \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} \qquad (3)$$

where generally: α, β, γ are the direction cosines of the rotated X'-, Y'- and Z'-axis relative to the original X-, Y-, Z-axes, respectively, and

$$\alpha_l\alpha_m + \beta_l\beta_m + \gamma_l\gamma_m = \delta_{lm} \qquad l,m = 1,2,3 \quad (4)$$

where δ_{lm} is the Kronecker delta.

In order to apply the Biot-Savart formula (1), we have to find suitable expressions $1_{ri}(\varphi)$ and $\vec{dl_i}(\varphi)$. By looking at Fig.1, if $X(\varphi)$, $Y(\varphi)$, $Z(\varphi)$ are the coordinates of the generic element $\overrightarrow{dl}_i(\varphi)$ and $1_x, 1_y, 1_z$ are rectangular unit vectors, we can obtain the three components of the magnetic flux density. For example, the x-component of B is in the form:

$$B_{xi}(x,y,z) = \frac{\mu_0 \underline{I}}{4\pi} \int_0^{2\pi L_i/h} \frac{dl_{yi}u_{rzi} - dl_{zi}u_{ryi}}{r_i^2} \qquad (5)$$

where with $k = h/2\pi$ e.g.:

$$u_{ryi} = \frac{y - \alpha_{2i} a\cos\varphi - \beta_{2i} a\sin\varphi - \gamma_{2i} k\varphi - y_i}{r_i} \qquad (6)$$

$$dl_{yi} = (-\alpha_{2i} a\sin\varphi + \beta_{2i} a\cos\varphi + \gamma_{2i} k)d\varphi \qquad (7)$$

$$\begin{aligned} r_i = [&(x - \alpha_{1i} a\cos\varphi - \beta_{1i} a\sin\varphi - \gamma_{1i} k\varphi - x_i)^2 \\ &+ (y - \alpha_{2i} a\cos\varphi - \beta_{2i} a\sin\varphi - \gamma_{2i} k\varphi - y_i)^2 \\ &+ (z - \alpha_{3i} a\cos\varphi - \beta_{3i} a\sin\varphi - \gamma_{3i} k\varphi - z_i)^2]^{1/2} \end{aligned} \quad . \qquad (8)$$

The integrals describing the magnetic flux density components have to be solved numerically.

The total magnetic field of the helical conductor with complex geometry can be obtained by superposition of the contributions produced by each segment.

It should be noted, that the formulas derived enables one to analyze magnetic fields produced by twisted-wire pair as well as by three-core cable considering the conductor twist. The twisted-pair cable can be represented mathematically as a double helix that consists of two helices having the same radius and pitch and carrying currents $\underline{I}$ and $-\underline{I}$; the helices are located 180 spatial degrees from each other. In the three-wire helix structure the current in the i-th conductor ($i = 1,2,3$) is $i_i = I\sqrt{2}\sin(\omega t + \psi_i)$ and the current phase angel $\psi_i = (i-1)2\pi/3$. The location of conductors in the $z = 0$ plane is fixed by angles φ_{0i}, where $\varphi_{0i} = (i-1)2\pi/3$. The total field components are found by summation.

II. EXAMPLE OF CALCULATIONS

In order to verify the correctness of the analytical calculations presented in the paper, comparison has been made with an analytical solution in form of infinite series containing Bessel functions obtained in [5] for infinitely long helical conductor. Fig.2 shows the B_φ component of the magnetic flux density along the axial direction at radial distance from helix axis r = 1 cm. The calculations have been curried out in central part of the helix for different helix length ranging from 1 m to 100 m, , and refer to I = 1 A, a = 1 mm, h = 2 cm and $\varphi_0 = 0$.

It follows from the calculations, that independent of helix length the agreement was excellent, for field components except of the B_φ, what is evident from physical point of view. Discrepancy between the results obtained by the use of the method appropriate for the infinitely long helical conductor and by use of the method for conductor finite in the length shows that for short helical conductors the proposed method shall be applied.

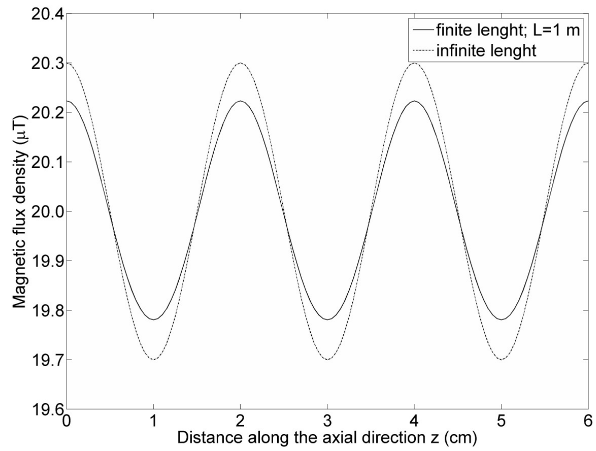

Fig.2. B_φ component versus z

III. FINAL REMARKS

The design of installation generating low-frequency magnetic field requires access to effective analytical and computational tools. The paper presents procedures of determining the magnetic flux densities intensities produced by currents in helical conductors with finite length basing on the Biot-Savart law. The analytical formulas for calculating the 3D magnetic field are derived and allow also managing cases with any complex geometry of the helical conductor such as changes of direction of a conductor line, changes of burial depth / height of the line and cables with twisted conductors as well.

The formulas allow tackling the magnetic field of the two-wire helix, as well as for the three-wire helix and can be used by a software tool to model the magnetic fields generated by e.g. twisted-wire pairs, twisted three-phase power cables, triplex service cables, helical coils, etc.

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